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Quantum logic and the question of the empiricality of logic

Abstract:

The article undertakes the issue of whether logic is empirical, that is, whether it is possible to modify logic due to empirical reasons. We will call this issue the problem of the empiricality of logic. Hilary Putnam's famous argument based on discoveries in quantum mechanics is discussed. Putnam claimed that classical logic should be discarded and replaced by quantum logic. This view is called the "Putnam's claim." It was met with harsh criticism from Dummett and Kripke, among others. While their criticism is largely valid, it does not dismiss the idea that logic itself could be empirical. The aim of this paper is to show that it is possible to present the discovery of the non-distributivity of quantum propositions as an empirical fact with implications for the revision of the adequacy of certain laws of logic, and thus, that empirical facts can have an impact on logic.

Keywords:

empiricality of logic, quantum logic, adoption problem

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Imagine that a group of scientists conducts a complicated experiment, the result of which proves beyond their comprehension. People in white lab coats gather in front of the control screen to see the string of inexplicable numbers. A murmur sweeps through the crowd, slowly growing as new explanatory hypotheses emerge. Is it a technical error? A computational one? An unprecedented anomaly? Hours pass, hypotheses multiply, but none proves to be correct.

– There is no other explanation, ladies and gentlemen – the project manager finally speaks. – We have simply refuted the laws of classical logic.

Does this situation appear absurd? After all, the laws of logic are the laws of reasoning, not the laws of nature. We consider them apriori truths that are universally and necessarily valid, regardless of experience.

Yet this was Hilary Putnam's claim: that logic is a discipline that can be revised on empirical grounds, and it is the empirical, and more specifically, quantum mechanics, that have provided us with reasons to revise it (Putnam, 1967).

Putnam's claim

In his famous 1967 article *Is logic empirical?* Putnam states that "logic is as empirical as geometry" (Putnam 1967, 184). And to what extent is geometry empirical? Well, sufficient enough for Euclidean geometry to be disproved as the geometry describing our physical world and for its place to be taken by the non-Euclidean geometry associated with the formalism of General Relativity (GR) (Putnam, 1967, 176). With the discovery of the curvature of space-time described by GR, it turned out that, contrary to our basic intuitions, lines which, from the point of view of Euclidean geometry, are not straight, can be considered straight (i.e., as the lines that are the shortest path from one point to another). And since GR has received excellent experimental confirmation, we have found ourselves in a situation where a proposition that only recently seemed unintuitive and impossible (i.e., that such a curved line is in fact straight) has turned out to be true (Putnam, 1967, 175).

According to Putnam, quantum mechanics has wrought upon logic a similar fate to the one which General Relativity wrought upon geometry (Putnam, 1967, 179). The issue boils down to the conflict, already noted by Birkhoff and von Neumann, between the quantum mechanical description of physical systems and the law of distributivity of conjunction and disjunction, which is a law of classical logic (Birkhoff, von Neumann, 1936, 10).

A characteristic feature of quantum mechanics is that it involves incompatible quantities, known as complementary or mutually exclusive quantities (Putnam, 1967, 180). An example of incompatible quantities are physical quantities which are tied by Heisenberg's uncertainty principle, such as, for example, selected components of position and momentum vectors. It is impossible to perform simultaneous measurements with arbitrary accuracy on such pairs of physical quantities. The consequence for quantum logic is that propositions such as *a)* "System *s* is at position *a*" and *b)* "The momentum of system *s* is *b*," cannot be simultaneously true, and consequently, their conjunction is *always* false. This is grounded in the mathematical formalism of quantum mechanics: the Hilbert subspaces representing the propositions *a* and *b* have such properties that when the operation corresponding to the conjunction of the propositions is performed on them, the empty space, which corresponds to falsity, is obtained as the result (Putnam, 1969, 179).

Now, suppose that it is known about our system *s* that its momentum is *b*, while about its position, it is known only that it can be located either in a region called *a*, or in a region that is not *a*, called a' (which is the orthocomplement of *a*). According to the law of distributivity, it should be true that

$$b \wedge (a \vee a') = (b \wedge a) \vee (b \wedge a'),$$

That is, the system has momentum *b* and is at position *a* or a' if and only if either it has momentum *b* and is at *a*, or it has momentum *b* and is at a'. However, we cannot say that this equality is true because of Heisenberg's uncertainty principle. If we assume that sentence *b* is true, and observe that the disjunction $a \vee a'$ is also true (because the subspaces representing *a* and a' sum to the whole space that represents the truth), then the conjunction on the left side is true. The right-hand side, in contrast, is not true, because position and momentum are incompatible quantities, thus, as we stated above, both conjunctions are false, and hence their disjunction is also false (Birkhoff, von Neumann, 1936, 10). Therefore, in the description of quantum systems, the law of distributivity fails.

The failure of distributivity is the characteristic feature of quantum logic. According to Putnam, this logic is derived from the Hilbert space itself, which is the foundation of the mathematical formalism of quantum mechanics (Putnam, 1969, 179). Moreover, Putnam believes that because quantum logic was not previously included in the description of quantum phenomena, many phenomena that are difficult to understand and explain have emerged. He discusses the cases of the complementarity of physical quantities, the superposition of states in a double-slit experiment, and the problem of measurement (Putnam, 1969, 183). According to Putnam, all of these difficulties can be solved in one stroke by simply rejecting the distributivity of logic (Putnam, 1969, 184). Moreover, according to him, this is a relatively minor change, that does not even interfere with the meaning of classical logical connectives (Putnam, 1969, 190). In other words, quantum mechanics combined with classical logic requires a nonstandard metaphysics that is not compatible with Putnam's scientific realism. On the other hand, quantum mechanics combined with quantum logic enables one to resolve quantum paradoxes and salvage a realistic metaphysics. The price is the law of distributivity, and Putnam is willing to pay it (Putnam, 1969, 189).

The Rise and Fall of the Analogy Between Logic and Geometry

Put in this way, Putnam's analogy between the change in geometry due to GR and the change in logic due to quantum mechanics appears to be quite clear. In the case of General Relativity, we face the following choice: on the one hand, we can retain Euclidean geometry, in which case, in order to describe the motion of bodies along the "curved lines" in this space (which are straight lines in the appropriate non-Euclidean geometry), additional "mysterious" forces have to be introduced; on the other hand, we can abandon Euclidean geometry, which is so intuitive to us, but then the description of motion will be simplified, we will obtain an explanation of a number of anomalies, and no mysterious forces will be needed to describe motion (Putnam, 1969, 191). The same is supposed to be the case with quantum mechanics: we can keep with classical logic, which we have never had any reason to doubt, but continue to face quantum paradoxes, or we can abandon it, and instead obtain an effective theory with a comprehensible, realistic metaphysics, in which all physical quantities are determined at all times (Putnam, 1969, 191).

Unfortunately, the analogy fades away when one considers that 1) Putnam's proposed solutions to paradoxes are not quite correct (Bell & Hallett, 1982, 357; Dummett, 1982, 273; Stairs, 2016, 25), 2) the rejection of the law of distributivity is found to interfere with the meaning of logical connectives (Bell & Hallett, 1982, 362; Dummett, 1976, 284), and 3) the absence of distributivity is incompatible with Putnam's realism (Dummett, 1976, 276). What's more, Dummet points out that if Putnam is to maintain simultaneously his own understanding of logical connectives and his realistic interpretation of quantum mechanics, his quantum connectives will work alongside classical ones, not *replace them*, "and this entails that the situation is not parallel to the geometrical case, and does not involve the abandonment, in response to experience or otherwise, of any logical law formerly held" (Dummett, 1976, 287).

It is at most an *expansion* of logic with some new non-classical connectives. On the other hand, if we were to replace classical logic with quantum logic but abandon the realistic interpretation of quantum mechanics, the argument for the empiricality of logic would be undermined because "it would no longer be possible to claim that the discovery of the invalidity of the distributive law was a discovery *about the world*" (Dummett, 1976, 288).

Putnam has continued his work on quantum logic for several years (Maudlin, 2018, 22). His main goal was to defend the realistic interpretation of quantum mechanics, which he characterized through value definiteness: the idea that 'a "measurement" merely reveals a pre-existing fact about a system, rather than playing a role in *creating* some new fact' (Maudlin, 2018, 24). This means that the measurement neither creates the measured observable during the measurement nor vests the system with a sharp value of a quantity which it did not possess beforehand. As is shown by various no-go theorems, such as the Kochen-Specker theorem, value definiteness cannot be upheld in quantum mechanics (Maudlin, 2018, 25; Bell & Hallett, 1982, 369; Stairs, 2016, 36). Various additional difficulties with this approach have led Putnam to abandon his attempts to treat quantum mechanics's difficulties with quantum logic (Maudlin, 2018, 38).

The issue of the empiricality of logic

Although Putnam's research program turned out to be a dead end, this does not yet disqualify the potential empiricality of logic. Perhaps the analogy with geometry doesn't hold up. But there still remains Birkhoff and von Neumann's argument for the non-distributivity of quantum propositions. The research tradition that grew out of the work of Birkhoff and von Neumann considers quantum logic to be a non-classical logic with algebraic semantics (usually) of (non-distributive) orthomodular lattices and rejects Putnam's idea of value definiteness. According to the above-mentioned opinion of Dummett, in this case, there should no longer be any question of the empiricality of logic, since, in his opinion, the rejection of distributivity is determined by considerations of the theory of meaning, not quantum mechanics (Dummett, 1982, 288). The thesis of replacing classical logic with quantum logic will neither be a proposition of quantum mechanics nor will it be derived from it (Dummett, 1982, 288). Instead, it will be drawn, according to Dummett, from the theory of meaning, i.e., from the search for the "correct model for the meanings which we confer upon our [...] statements" (Dummett, 1982, 288).

Kripke, in turn, believes that the very idea that logic could be revised due to empirical factors is inherently contradictory (Kripke, 2024; Stairs, 2016, 29). Kripke first draws a distinction between *logic* and *systems of logic*. He understands the former as principles of reasoning that we all, spontaneously as it were, follow, while systems of logic are formal creations that aim to approximate, for better or worse, logical reasoning (Kripke, 2024, 33). While there are many systems of logic, there is only one logic. There are not multiple logics to choose from. Among other things, logic is used to make intuitive judgments about which formal systems are "correct" (Stairs, 2016, 29). Kripke believes that sometimes a particular system of logic receives an interesting informal interpretation, which leads us to believe that we are dealing with some laws of logic. Sometimes it may also happen that we discover some new connectives and identify the laws of logic pertaining to them (Kripke, 2024, 33). It may also be the case that we discover that a formula we believed to be a law of logic is in fact not one. However, according to Kripke, this is not an adoption of a new logic but rather a discovery about a particular system of logic (Kripke, 2024, 33). And what's more, according to Kripke, all such discoveries take place through *reasoning*, not empirical investigation (Kripke, 2024, 34).

However, as Stairs notes, even if we accept Kripke's distinction between logic and systems of logic, this does not preclude that sometimes the question of assessing the adequacy of particular laws or systems may sometimes depend on what the world is actually like, i.e. on empirical issues. Stairs gives as one example the discussion of future contingency, determinism and presentism (Stairs, 2016, 33). A second possible example could be quantum logic (Stairs, 2016, 35). How would it work?

How could the empirical possibly have a bearing on logic?

We will begin by recalling that logic studies certain relations that occur in language. For the purpose of these studies, there are constructed models of natural language or its fragments, which have two basic components: syntax and semantics (Beall & van Fraassen, 2003, 23). Syntax defines the structure of a language: it specifies its alphabet, that is, the basic symbols, and provides the rules for forming well-formed formulas, that is, its grammar. Semantics, on the other hand, provides admissible valuations of the formulas, that is, functions which assign to them certain values, such as truth or falsity. A language consists of its syntax and a non-empty class of its valuations. The role of logic is to determine which propositions of the language are logically true and which inferences are valid. A system of logic defines a syntactically defined relation of consequence (the derivability of a conclusion from a set of premises). When this relation coincides with a semantically defined consequence (logical entailment), the system is strongly complete with respect to a given language. A system of logic can be strongly complete with respect to many languages (van Fraassen, 1979, 139).

Semantics bridges the gap between systems of logic and the world: by providing propositions with a logical value and imposing constraints on the class of admissible valuations, it introduces the possibility of evaluating inferences not only in terms of validity but also in terms of whether and what can be said about the world in this language. On the other hand, we can also modify the semantics to conform to the fragment of the world we want to model and for which we want to find an appropriate system of logic.

In classical propositional calculus, admissible valuations assign the formulas values from the set {0, 1}, observing some specific rules, such as that the formulas φ and ¬φ cannot have the same logical value. 1 and 0 are interpreted as truth and falsity, respectively. It turns out that the classical propositional calculus is also complete with respect to languages in which valuations take as values the elements of a two-valued matrix, an algebra of sets or a Boolean algebra. These semantics are referred to as the matrix semantics, set-theoretic semantics, and algebraic semantics, respectively. Quantum logic is not complete with respect to any of these languages. Instead, it is complete with respect to the class of (non-distributive) orthomodular lattices.

One might say that the classical propositional calculus is appropriate for those objects that can be treated as elements of a set, so that the logical connectives would behave analogously to operations on sets. But as it turns out, there exist some objects that do not behave in this manner. Namely, the states of quantum systems at the microscopic level behave like vectors; one can perform vector operations on them, but not set operations. This is particularly evident in the case of the disjunction, which in quantum logic is interpreted as the supremum of a subspace (vector addition), and in classical logic as the set union. The discovery that micro objects behave in such a way has, of course, both a theoretical and an empirical component: formulating a theory is an intellectual work that is not based solely on empirical evidence; but it is a matter of empirical work, i.e. testing of the theory, to determine whether or not it describes the world correctly. At the moment, in the case of quantum mechanics, we have no doubt that the theory is effective and correctly describes the behavior of micro objects.

Hence, it seems to be justified to say that empirical tests have confirmed that there is a class of physical objects (in the micro-world) for the description of which the Hilbert space theory is more appropriate than set theory. Consequently, we have found that there is a realm of the world in which it is more appropriate to use logic with the semantics of orthomodular lattices rather than set theory.

What does this mean for classical logic? It certainly doesn't mean that it has been "refuted" or that we should replace it. If it is the case, as Kripke argues, that there are various systems of logic which approximate different parts of logic as such, then it is enough to conclude that we have discovered a new domain where a certain law, which we have hitherto considered a law of logic, cannot be applied, even if it can be applied elsewhere. Such a conclusion is consistent with the position of local pluralism, according to which different logics apply to different areas of discourse (Czernecka-Rej, 2014, 79; Haack, 1978, 221).

We said that logic studies certain relations occurring in language. Language, in turn, informs about the world. The world certainly influences the evaluation of the premises in inferences. However, it is not inconceivable that the kinds of inferences we consider valid are derived from the kinds of objects we interact with on a daily basis. If this is the case, the discovery of a new, unintuitive type of objects could lead us to reconsider how we should think and reason about them.

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